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Topology optimization of building with stochastic wind loads

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SUMMARY:
This paper considers topology optimization of wind-excited structures, where the random wind field is modeled as a filtered vector white noise. An augmented state space representation is formed combining the equation of motion for the structure with the excitation model, and the stationary covariances of the structural responses of interest are then obtained by solving the associated Lyapunov equation. Objective functions for the optimization scheme are defined in terms of the stationary covariance of the response. The proposed topology optimization scheme is illustrated for a tall building subjected to across-wind loads. The results presented herein demonstrate the efficacy of the proposed approach for efficient topology optimization of buildings subjected to stochastic wind excitation.

Keywords: Topology optimization, stochastic dynamics, Lyapunov equation

1. INTRODUCTION

Topology optimization provides a general approach to obtain optimal material layout in a structural domain according to a given objective function and constraints (Bendsøe and Sigmund 2003). Several researchers have conducted topology optimization for wind-excited buildings, by approximating the wind load as static lateral loads; this simplified approach produces designs that are dependent on the spatial discretization of the wind load and does not account for the stochastic dynamic nature of the wind load (Simiu and Scanlan 1996). Recently, Gomez et al. (2021) proposed a method to model random wind fields as a filtered vector white noise which can be integrated efficiently into the topology optimization framework proposed by Gomez and Spencer (2019) for topology optimization of structures subjected to stochastic dynamic loading. This paper presents a topology optimization framework for buildings subjected to stochastic wind excitation. An augmented state space representation is formed by combining the equation of motion for the structure with the wind excitation filter. Then, the stationary covariances of the structural responses of interest are obtained by solving a Lyapunov equation and used to form the objective function. To demonstrate the efficacy of the proposed approach, optimization of the bracing system in tall buildings to stochastic across wind excitation is presented.

2. PROBLEM FORMULATION

This section briefly summarizes the formulation for topology optimization of buildings subjected to stochastic wind load. Further details can be found in the study by Gomez et al. (2021).

2.1. Stochastic wind load and structural model

The stochastic wind excitation is modeled using the following state space representation

$$
\begin{aligned}
\dot{\mathbf{x}}_{\mathrm{f}} &= \mathbf{A}_{\mathrm{f}}\mathbf{x}_{\mathrm{f}} + \mathbf{B}_{\mathrm{f}}\mathbf{w}(t) \\
\mathbf{f} &= \mathbf{C}_{\mathrm{f}}\mathbf{x}_{\mathrm{f}}
\end{aligned}
\tag{1}
$$

where **f** is a multivariate discretization of the wind field along the height of the building, and the state matrices and state vector are obtained by the method proposed by Gomez et al. (2021). The input of the model is a multivariate white noise **w**. On the other hand, the structural system is represented by the state space form

$$
\begin{aligned}
\dot{\mathbf{x}}_{\mathrm{s}} &= \mathbf{A}_{\mathrm{s}}\mathbf{x}_{\mathrm{s}} + \mathbf{B}_{\mathrm{s}}\mathbf{f}(t) \\
\mathbf{y} &= \mathbf{C}_{\mathrm{s}}\mathbf{x}_{\mathrm{s}} + \mathbf{D}_{\mathrm{s}}\mathbf{f}(t)
\end{aligned}
\tag{2}
$$

Combining both systems, the augmented system is obtained with state space representation

$$
\begin{aligned}
\dot{\mathbf{x}}_{\mathrm{a}} &= \mathbf{A}_{\mathrm{a}}\mathbf{x}_{\mathrm{a}} + \mathbf{B}_{\mathrm{a}}\mathbf{w}(t) \\
\mathbf{y} &= \mathbf{C}_{\mathrm{a}}\mathbf{x}_{\mathrm{a}}
\end{aligned}
\tag{3}
$$

where $\mathbf{y}$ is the output considered in the objective function of the optimization. The covariance of the stationary response can be obtained by solving the Lyapunov equation

$$
\mathbf{A}_{\mathrm{a}}\boldsymbol{\Gamma}_{\mathbf{x}_{\mathrm{a}}} + \boldsymbol{\Gamma}_{\mathbf{x}_{\mathrm{a}}}\mathbf{A}_{\mathrm{a}}^{\mathrm{T}} + 2\pi\mathbf{B}_{\mathrm{a}}\mathbf{S}_0\mathbf{B}_{\mathrm{a}}^{\mathrm{T}} = \mathbf{0}, \quad \text{and} \quad \boldsymbol{\Gamma}_{\mathbf{y}} = \mathbf{C}_{\mathrm{a}}\boldsymbol{\Gamma}_{\mathbf{x}_{\mathrm{a}}}\mathbf{C}_{\mathrm{a}}^{\mathrm{T}}
\tag{4}
$$

2.2. Harmful and harmless interstory drifts

An important measure of building performance is often given in terms of the interstory drift, which is defined as the relative lateral displacement occurring between two consecutive floors. Zhou et al. (2012) proposed to decompose interstory drift of the i^{th} story, Δu_i, into harmless and harmful components, i.e., $\Delta u_i = d_i^{\mathrm{HL}} + d_i^{\mathrm{HF}}$. The harmless interstory drift, d_i^{HL}, is related to the rigid body rotation from the lower floor and the harmful drift, d_i^{HF}, also known as shearing drift, refers to the shear component that directly causes damage to the story. The harmless drift is computed as the projected displacement due to the rotation of the previous story. In this study, the floor rotation, θ_{i-1}, is estimated using the tangent method with the vertical displacements of the previous floor (Zhou et al. 2012).

2.2. Topology optimization formulation

The design variables in continuous-domain topology optimization are chosen as the relative density in each element (Bendsøe and Sigmund 2003); for the element n, the relative density variable is denoted by z_n. The problem is then to find **z** and β such that:

$$
\begin{aligned}
\min_{\mathbf{z},\beta} \quad & \beta + \alpha^2 J_{\mathrm{RD}} \\
\text{s.t.} \quad & J_i(\mathbf{z}) - \beta \le 0 \quad \text{for } i = 1,2,\dots,N_{\mathrm{f}} \\
& g(\mathbf{z}) = V(\mathbf{z}) - V_{\max} \le 0 \\
& z_n \in [z_{\min}, z_{\max}] \text{for } n = 1,2,\dots,N_{\mathrm{el}}
\end{aligned}
\tag{5}
$$

where J_i represents the variance of the harmful interstory drift of the i^{th} story and J_{RD} represents the variance of the of roof displacement.

The optimization problem is solved by using the method of moving asymptotes (Svanberg 1987). To obtain the sensitivities, adjoint Lyapunov equations are solved to obtain the Lagrange multipliers and the sensitivities of the objective function and constraints. To avoid mesh-dependency and numerical instabilities a filter is applied (Bendsøe and Sigmund 2003).

3. ILLUSTRATIVE EXAMPLE

A 76-story four-bay building is considered, where the general properties are taken from a benchmark in structural control (Yang et al. 2004). The two perimeter braced frames, to be designed, are assumed to act as the lateral resisting system for the wind load in that direction, and the secondary moment frame gravity system is included and is coupled with the bracing system at column-beam joints. The height of each story is 4.03 m and the building has 4 bays with a span equal to 10.5 m each, which define the gravitational or secondary system. Two lumped masses of 2×10^5 kg are located at each floor and are located at each boundary. A flexible diaphragm is considered at each floor. The following parameters are used for the stochastic across-wind model proposed by Chen (2014): $\eta = 1/3$, $U_r = 15$ m/s, $C_z = 5$, $\sigma_L = 0.4$, $S_t = 0.07$, $\beta = 0.5$.

The braced system is going to be designed to minimize the response to stochastic across-wind excitation. The design domain is given by a 42 m×306 m rectangle, which is composed of a solid linear elastic material having the following properties: Young's modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.3$, density $\rho = 7500$ kg/m^3, and Ersatz parameter $\epsilon = 10^{-4}$. The domain is assumed to have a uniform thickness of 0.2 m; and the continuum domain is assumed to be in plane stress condition and is discretized using 52×380 Q4 elements. The radius of the filter is equal to 1.50 m. The total volume of the optimization variables is constrained to be less or equal than 0.30 of the solid domain.

Topology optimization is performed to minimize the maximum interstory drift variance among all stories (MAX), roof displacement variance (RD), and combined response (Comb.) of the MAX and RD single-objectives with $\alpha=10^{-3}$. Fig. 1 shows the results for these three different objectives functions.

The resulting topologies are composed of sets of braces that span several floors. The topology for the multi-objective function is an intermediate result between the two single-objective results. The MAX objective yields approximately a uniform distribution of harmful interstory drifts, with the maximum value being considerably smaller than the maximum value in the RD design. The RD design achieves smaller total interstory drifts, roof displacements, and accelerations than the MAX design. In all cases, the maximum floor acceleration occurs at the roof. The multi-objective design achieves a maximum harmful drift and roof displacement that are only slightly larger than the corresponding single-objective designs; so, it improves the fallbacks of both designs. The multi-objective design yields good local and global performance.

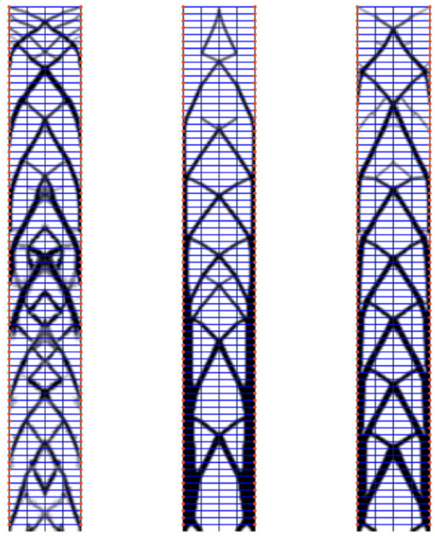

Figure 1. Topologies for the stochastic across-wind excitation minimizing (a) maximum harmful interstory drift, (b) roof displacement, (c) combined response.

4. CONCLUSIONS

This paper implemented an efficient topology optimization framework for buildings subjected to stochastic wind excitation. The roof displacement, the maximum harmful interstory drift, and combined response are used as objective functions; the harmful interstory drift is considered because it is the shearing component of the drift that causes damage in the story. A volume constraint was imposed to limit the design space, and the design variables were chosen as the relative densities in each element, which were bounded to achieve physically meaningful solutions. The example optimized the lateral resisting system for across-wind stochastic load. Due to the trade-off in the design, a multi-objective design was performed by combining both objectives, and the corresponding design achieves a performance with a good balance between both responses. The results presented herein demonstrate the efficiency of the proposed approach for topology optimization of buildings excited by wind, which presents a useful tool for designers to explore new types of structural patterns for tall buildings.

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